8.WPGI2024

Estimating Hardening Soil-Brick model parameters for sands using CPTU tests

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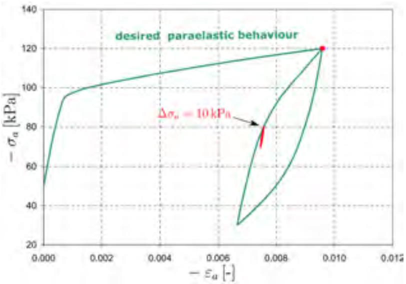
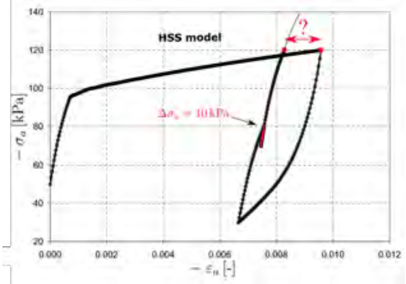
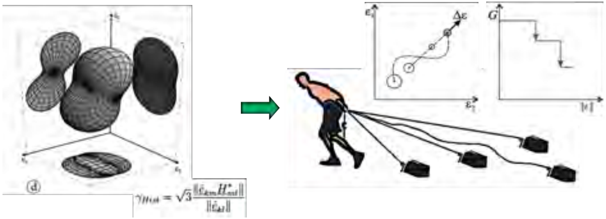
Hardening Soil-small vs Hardening Soil-Brick





HSs → HS-Brick

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Refinement of the Hardening Soil model within the small strain range

Research Paper | Open access | Published: 13 March 2020

Volume 15, pages 2031–2051, (2020) Cite this article

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Marcin Cudny ✉ & Andrzej Truty

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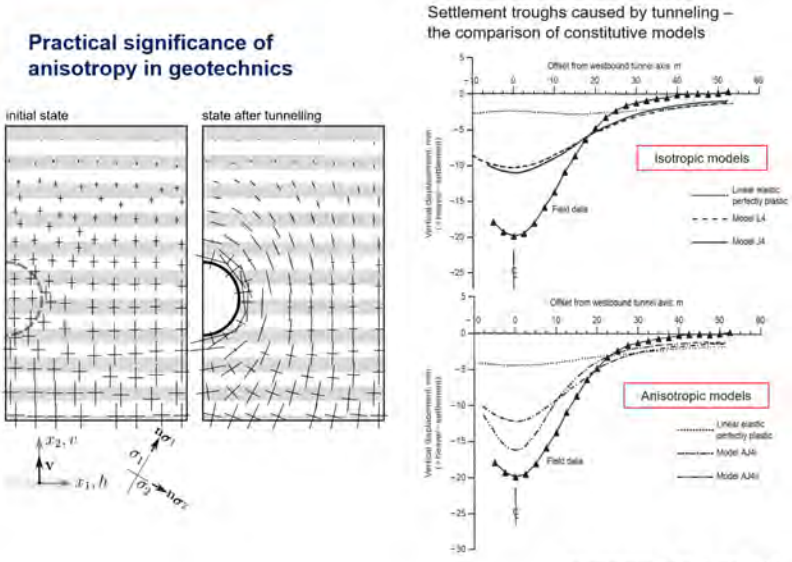
Niemunis, A. and Cudny, M. (2018). “Discussion on ”Dynamic soil-structure interaction: A three- dimensional numerical approach and its application to the Lotung case study”. Poor performance of the HSS model.” Computers and Geotechnics, 98, 243–245.

Summary of HS-Brick model parameters (isotropic version)

- Stiffness: E_0^{ref}, $\gamma_{0.7}$, $E_{\text{ur}}^{\text{ref}}$, E_{50}^{ref}, $E_{\text{oed}}^{\text{ref}}$, m, ν
- Stiffness barotropy: $E_0 = E_0^{\text{ref}} \left(\frac{p'}{p_a} \right)^m$ or $E_0 = E_0^{\text{ref}} \left(\frac{\sigma_3}{p_a} \right)^m$
- Strength: ϕ'_{peak}, c'
- Dilatancy: ψ
- Stress history: $\text{OCR}(z) \Rightarrow K_0$
- In sandy soils it is safe to assume $OCR = 1$, this assumption yields $K_0 = K_0^{\text{NC}}$ (use Jaky formula here)

Anisotropic AHS-Brick model

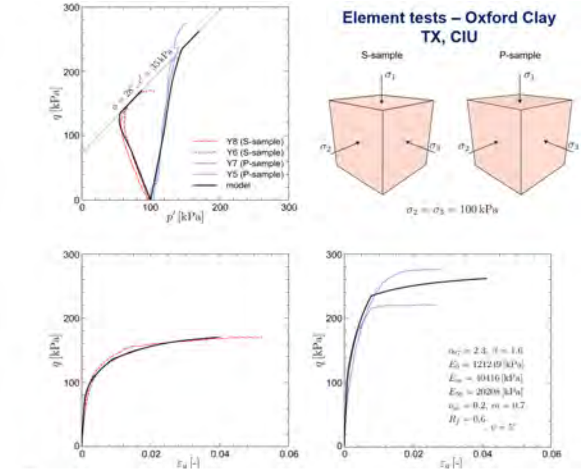


Practical significance of anisotropy in geotechnics

Settlement troughs caused by tunneling – the comparison of constitutive models

Addenbrooke, Potts, Puzrin, 1997

Anisotropic AHS-Brick model



Jeden dodatkowy parametr $\Rightarrow \alpha_G = \dfrac{G_{hh}}{G_{hv}}$

K. Lisewska, M. Cudny and A. Truty. Incorporation of cross-anisotropic small-strain stiffness into the Hardening Soil-Brick model. Korbielów 2024.

Is it possible to estimate HS-Brick model parameters for young uncemented sands from the CPTU tests only?

1. Laboratory tests on sands are difficult as samples have to be usually reconstituted
2. Without the evidence deduced from laboratory tests (high quality triaxial and oedometer tests) answer is ⇒ **NO**
3. With a certain laboratory experience it is definitely possible but with limitation to the uncemented sands only (so far..)
4. This research has been made based on experiments conducted by Wichtmann et al. but also on experiments conducted on coarse and medium sands from Łódź and Warsaw (Poland)

Wichtmann, T., Kimming, I., and Traintafyllidis, T. (2017). “On correlations between “dynamic” (small-strain) and “static” (large-strain) stiffness moduli – an experimental investigation on 19 sands and gravels.” Soil Dynamics and Earthquake Engineering, 98, 72–83.

Truty, A. Estimating Hardening Soil-Brick model parameters for sands based on CPTU tests and laboratory experimental evidence. Scientific Reports 14, 15102 (2024). https://doi.org/10.1038/s41598-024-65789-5

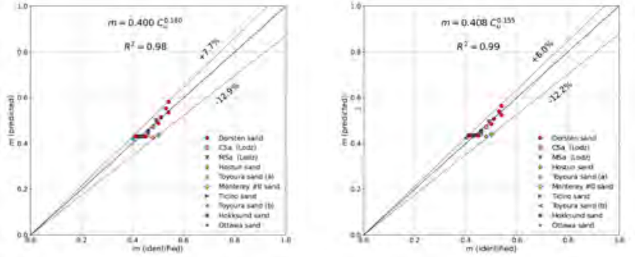
Parameter: m

- m parameter is defined here for the range of very small strains
- Strong correlation between m and $C_u = d_{60}/d_{10}$ has been found by Wichtmann
- Wichtmann (2009): $m = 0.4\ C_u^{0.18}\,(1 + 0.116\ \ln(1 + 100\ \mathrm{FC}))$
- Truty (2024): $m = 0.408\ C_u^{0.155}\,(1 + 0.092\ \ln(1 + 100\ \mathrm{FC}))$
- **Remark: m is increasing with shear strain; what can we do with that ?**

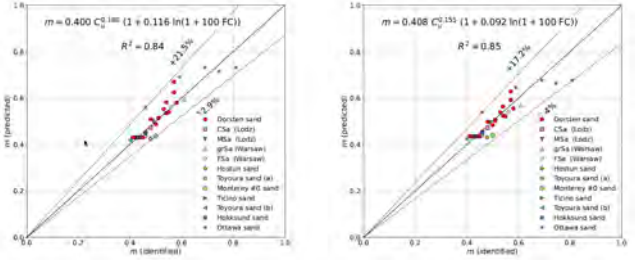
Truty, A. Estimating Hardening Soil-Brick model parameters for sands based on CPTU tests and laboratory experimental evidence. Scientific Reports 14, 15102 (2024). https://doi.org/10.1038/s41598-024-65789-5

Parameter: *m*

Clean sands

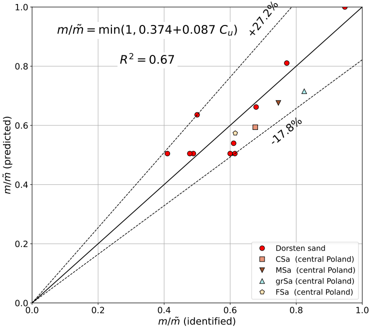


All sands



Parametr: $\tilde{m}$

- Exponent m is an increasing function of the equivalent shear strain
- Here $\tilde{m}$ corresponds to the E_{50} modulus
- Truty (2023): $m/\tilde{m} = 0.374 + 0.087\ C_u \leqslant 1$



Relation between G_0^{ref} and E_{50}^{ref}

- **Constraint**: $\dfrac{E_0^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}} = \underbrace{\dfrac{E_0^{\mathrm{ref}}}{E_{\mathrm{ur}}^{\mathrm{ref}}}}_{>1} \underbrace{\dfrac{E_{\mathrm{ur}}^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}}}_{>2} > 2$
- **Hence**: $\dfrac{G_0^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}} > \dfrac{1}{1+\nu}$
- **More restrictive form**: $\dfrac{G_0^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}} > 1$
- **Proposed approximation**: $\dfrac{G_0^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}} = 1 + A \left(E_{50}^{\mathrm{ref}}/p_a\right)^n$

Wichtmann, T., Kimming, I., and Traintafyllidis, T. (2017). "On correlations between "dynamic" (small-strain) and "static" (large-strain) stiffness moduli – an experimental investigation on 19 sands and gravels." Soil Dynamics and Earthquake Engineering, 98, 72–83.

Relation between G_0^{ref} and E_{50}^{ref}: coarse and medium sands

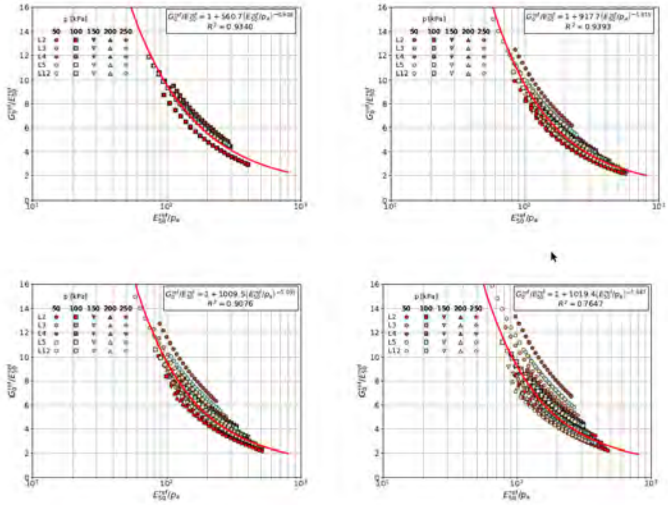


Fig. 5 Relations $G_0^{\mathrm{ref}}/E_{50}^{\mathrm{ref}}$ vs $E_{50}^{\mathrm{ref}}/p_a$ for coarse and medium sands (in the top left figure distinct m and $\bar{m}$ values are used, in the top right figure $\bar{m} = m$ is assumed, in the bottom left figure fixed $m = \bar{m} = 0.5$ is assumed, in the bottom right figure the averaged value $0.5(m + \bar{m})$ is used

Relation between G_0^{ref} and E_{50}^{ref}: verification for Łódź sands

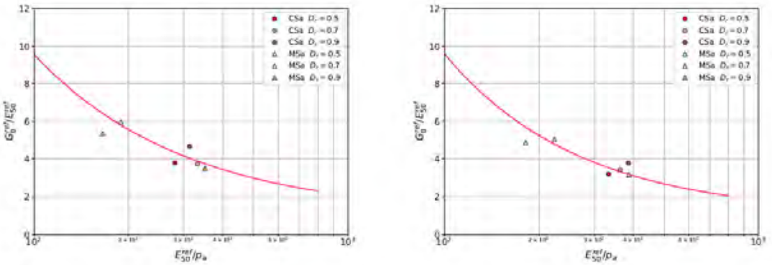
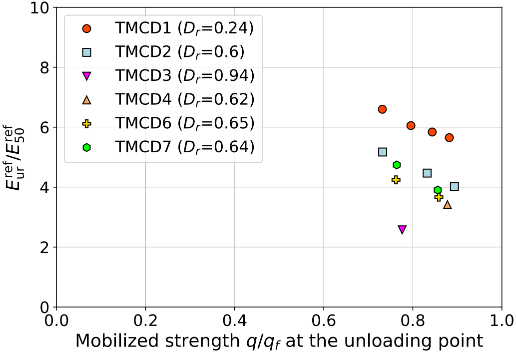
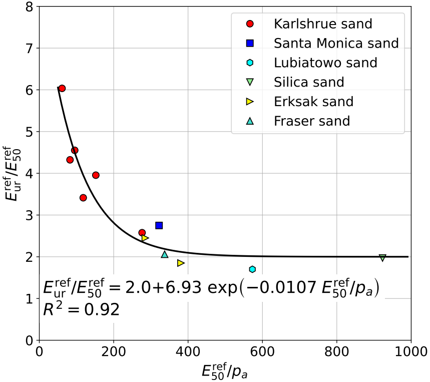


Fig. 8 Verification of derived relations $G_0^{\mathrm{ref}}/E_{50}^{\mathrm{ref}}(E_{50}^{\mathrm{ref}}/p_a)$ for coarse and medium Łódź sands. In the left figure distinct m and $\tilde{m}$ values are used for the two moduli while in the right plot enforced $\tilde{m} = m$ is used

Relation between $E_{\mathbf{ur}}^{\mathrm{ref}}$ and E_{50}^{ref}

- Results of experiments conducted by Wichtmann shown in the left figure
- Reloading has to begin from relatively high $q/q_f > 0.8$ while unloading must be continued till $q \approx 0$





Relation between $E_{\mathbf{oed}}^{\mathrm{ref}}$ i E_{50}^{ref}: medium and coarse sands

1. Approximation: $\dfrac{E_{\mathrm{oed}}^{\mathrm{ref}}}{E_{50}^{\mathrm{ref}}} = A \left(E_{50}^{\mathrm{ref}}/p_a\right)^n$
2. Here we have to be careful when choosing the reference vertical stress hence $\sigma_{\mathrm{v,oed}} = 8\ p_a$
3. For very low or very high $E_{50}^{\mathrm{ref}}/p_a$ values model may still not be able to reproduce this relation

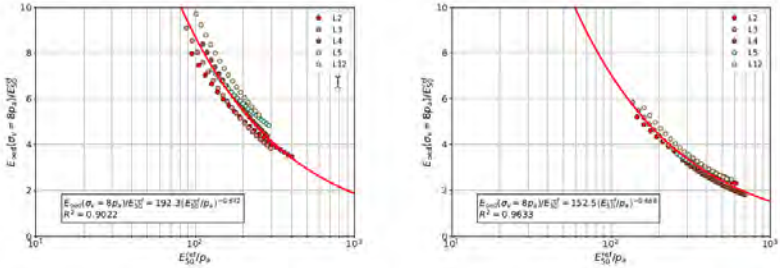


Fig. 13 Relation between $E_{\mathrm{oed}}^{\mathrm{ref}}\ (\sigma_v = 8\ p_a)\,/E_{50}^{\mathrm{ref}}$ and $E_{50}^{\mathrm{ref}}/p_a$ for medium and coarse sands (in the left figure E_{50}^{ref} modulus computed using exponent $\bar{m}$ while in the right one exponent m respectively)

Wichtmann, T., Kimming, I., and Traintafyllidis, T. (2017). "On correlations between "dynamic" (small-strain) and "static" (large-strain) stiffness moduli – an experimental investigation on 19 sands and gravels." Soil Dynamics and Earthquake Engineering, 98, 72–83.

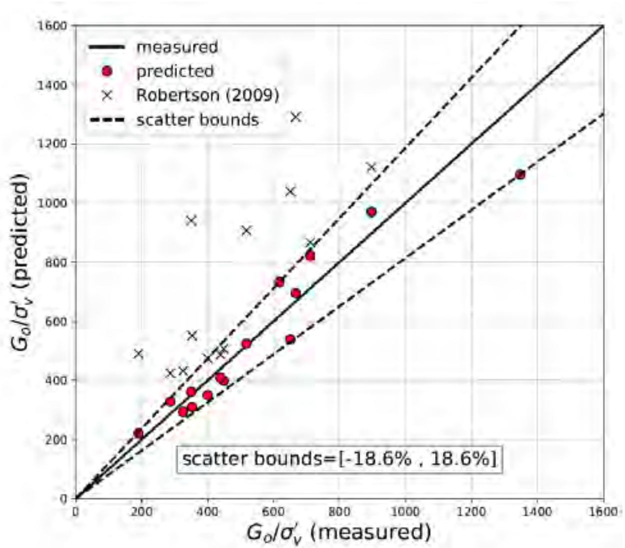
Estimating E_0^{ref} using CPTU test

- Robertson's formula for v_s yields usually too high G_0 values
- Moreover it does not reproduce the power law in barotropy function $G_0 - p$ (Ahmed 2017)
- From experiments we know that: $\dfrac{G_0}{p_a} \propto \left(\dfrac{p'}{p_a}\right)^m$
- Hence: $\dfrac{G_0}{p_a} \propto \left(\dfrac{\sigma'_v}{p_a}\right)^m$ as $p' = \dfrac{1 + 2\ K_0}{3}\sigma'_v$
- For the data base of 15 sands published by Mayne the following corelation formulas were derived by Ahmed (2017)
  - $m = 0.167\ I_c - 0.002\sigma'_v/p_a + 0.232$
  - $G_0 = 5300\ \sigma'_v\ \exp\left(-1.25\ I_{cc}\right) f(F_r)$ (reported estimation error +/-20%)
  - $f(F_r) = 0.21\ F_r + 0.85 \geqslant 1.0$
  - Corrected behavioral index $I_{cc} = \sqrt{\left(3.47 - \log Q_{tc}\right)^2 + \left(1.22 + \log F_r\right)^2}$
  - $Q_{tc} = \dfrac{Q_{tn}}{\left(\sigma'_v/p_a\right)^{1-m}}$

Ahmed, S. M. (2017). "Enhancing the CPT correlation with the small strain shear stiffness of sands." Ain Shams Engineering Journal, 8, 539–548.

Estimating E_0^{ref} using CPTU test

- Robertson's formula for v_s yields usually too high G_0 values



Estimating E_0^{ref} using CPTU tests

- Correcting Ahmed's formula (using particle swarm optimization)
  - $m = 0.111\ I_c + 0.322$
  - $G_0 = 7127\ \sigma'_v\ \exp\left(-1.432\ I_{cc}\right)\ f(F_r)$
  - $f(F_r) = 0.343\ F_r + 0.812$

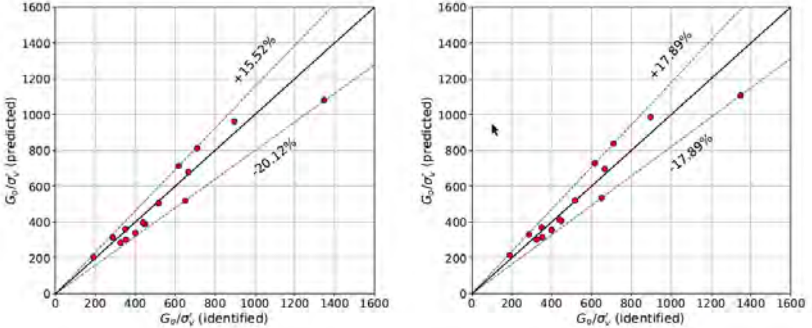
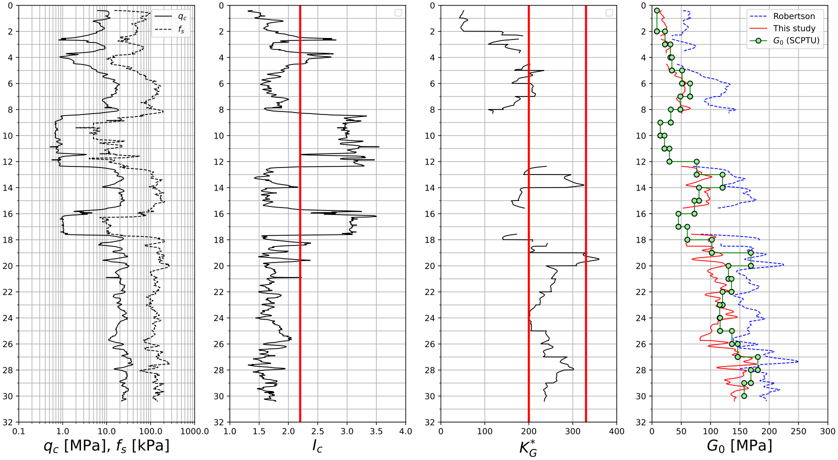


Fig. 15 Predicted vs measured G_0/σ'_v values for 15 sands from data base by [18] (results of unconstrained and constrained optimization are shown in the left and right figure respectively)

How this formula works in the practice?

SCPTU (Gdańsk)



Stiffness degradation in the range of small strains

- Wichtmann et al. proposed the following formula for the stiffness degradation

$$\frac{G}{G_0} = \frac{1}{1 + a\dfrac{\gamma}{\gamma_r}}$$

where

$$\begin{aligned} a &= \left(1 + 0.847 \ln\left(C_u\right)\right) \exp\left(2.05\ \mathrm{FC}\right) \\ \gamma_r &= \frac{\tau_{\max}}{G_0} \end{aligned}$$

- The maximum shear stress at in situ conditions can be expressed as follows

$$\tau_{\max} = \max\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \frac{\sigma_v'(1 - K_a)}{2}$$

T. Wichtmann and T. Traintafyllidis. Effect of uniformity coefficient on G/Gmax and damping ratio of uniform to well graded quartz sands. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 139(1):59–72, 2013.

T. Wichtmann, M. A. Navarette Hernandez, and T. Traintafyllidis. On the influence of a non-cohesive fines content on small strain stiffness, modulus degradation and damping of quartz sand. Soil Dynamics and Earthquake Engineering, 69: 103–114, 2015

Stiffness degradation in the range of small strains

- In the HS-Brick model the adopted stiffness degradation curve is defined as follows

  $$\frac{G}{G_0} = \frac{1}{1 + \tilde{a}\dfrac{\gamma}{\gamma_{0.7}}}$$

  where $\tilde{a} = 0.385$ is assumed.

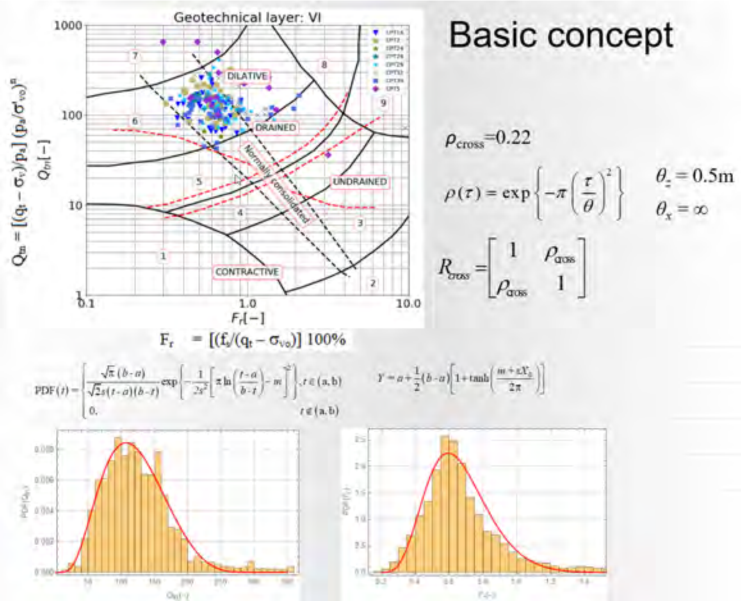
- By comparing the two expressions (Wichtmann's formula and the one adopted in the HS-Brick model) the following formula for the parameter $\gamma_{0.7}$ is derived (FC in decimal notation)

  $$\gamma_{0.7} = \frac{\tilde{a}}{a}\gamma_r = \frac{0.385}{(1 + 0.847\ln(C_u))\exp(2.05\,\mathrm{FC})}\,\frac{\sigma_v'(1 - K_a)}{2G_0}$$

- Note that $\gamma_{0.7}$ varies with p'; hence we could adjust it according to the in situ stress conditions

- C_u can be obtained form the CPTU using inverse $m - C_u$ relation

Prelude to random fields/variables



Basic concept

$$\rho_{cross} = 0.22$$

$$\rho(\tau) = \exp\left\{-\pi\left(\frac{\tau}{\theta}\right)^2\right\} \qquad \theta_z = 0.5\mathrm{m} \quad \theta_x = \infty$$

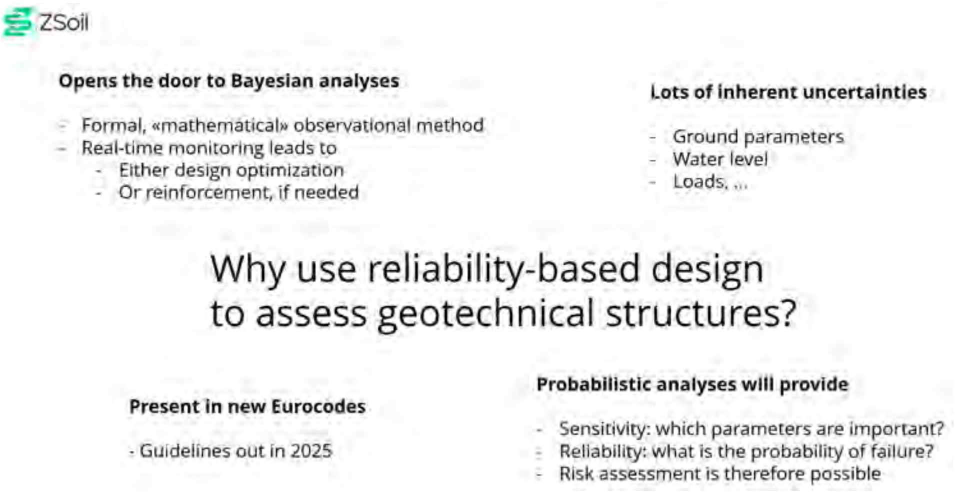
$$R_{cross} = \begin{bmatrix} 1 & \rho_{cross} \\ \rho_{cross} & 1 \end{bmatrix}$$

$$F_r = [(f_s/(q_t - \sigma_{vo})] \, 100\%$$

$$\mathrm{PDF}(t) = \begin{cases} \dfrac{\sqrt{\pi}(b-a)}{\sqrt{2}s(t-a)(b-t)} \exp\left\{-\dfrac{1}{2s^2}\left[\pi \ln\left(\dfrac{t-a}{b-t}\right) - m\right]^2\right\}, & t \in (\mathrm{a,b}) \\ 0, & t \notin (\mathrm{a,b}) \end{cases}$$

$$Y = a + \frac{1}{2}(b-a)\left[1 + \tanh\left(\frac{m + sX_0}{2\pi}\right)\right]$$

Reliability based design



ZSoil

Opens the door to Bayesian analyses

- Formal, «mathematical» observational method
- Real-time monitoring leads to
    - Either design optimization
    - Or reinforcement, if needed

Lots of inherent uncertainties

- Ground parameters
- Water level
- Loads, ...

Why use reliability-based design
to assess geotechnical structures?

Present in new Eurocodes

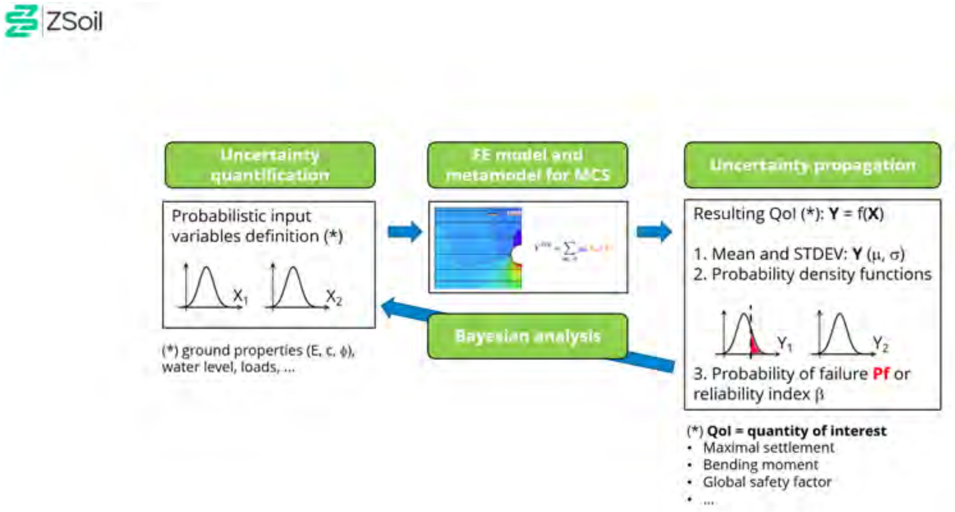
- Guidelines out in 2025

Probabilistic analyses will provide

- Sensitivity: which parameters are important?
- Reliability: what is the probability of failure?
- Risk assessment is therefore possible

Slide from presentation by Stephane Commend HEIA Fribourg CH

Reliability based design



ZSoil

Uncertainty quantification

FE model and metamodel for MCS

Uncertainty propagation

Probabilistic input variables definition (*)

(*) ground properties (E, c, ϕ), water level, loads, ...

Resulting QoI (*): **Y** = f(**X**)

1. Mean and STDEV: **Y** (μ, σ)
2. Probability density functions

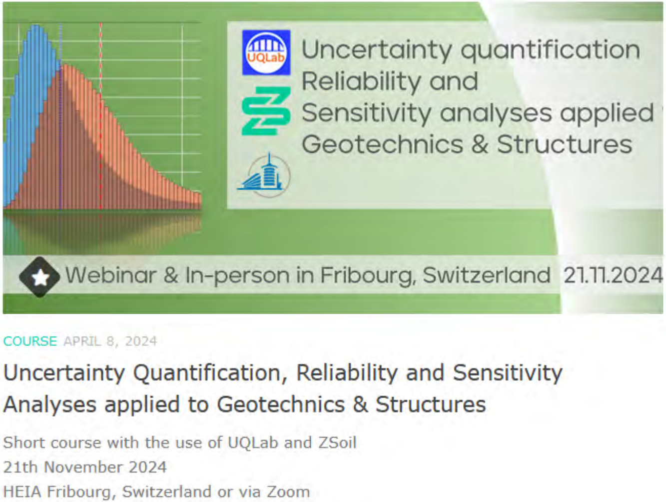
3. Probability of failure **Pf** or reliability index β

Bayesian analysis

(*) **QoI = quantity of interest**
- Maximal settlement
- Bending moment
- Global safety factor
- ...

Slide from presentation by Stephane Commend HEIA Fribourg CH

Short course on uncertainty quantification



COURSE APRIL 8, 2024

Uncertainty Quantification, Reliability and Sensitivity Analyses applied to Geotechnics & Structures

Short course with the use of UQLab and ZSoil
21th November 2024
HEIA Fribourg, Switzerland or via Zoom

Wejście do rejestracji ze strony www.zsoil.com